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Synergetic Representation of Stick–Slip Mode of Boundary Friction

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Abstract—The melting of an ultrathin lubricating film clamped between two atomically smooth solid surfaces that are in relative motion is studied based on the Lorentz model for the approximation of a viscoelastic medium. An equation of motion for the stresses has been derived in the form of a three-order differential equation and analyzed at various friction surface temperatures. In all cases, the phase portraits and the time dependences of the stresses have been plotted. It has been found that, depending on the temperature and the lubricant parameters, either the damped oscillation mode or the stochastic oscillation mode may occur. The stochastic oscillation mode is presented in the phase plane as a strange attractor. It has been shown that initial conditions have a critical effect on the system behavior. Based on the model, the behavior of two types of tribosystems, i.e., with the unidirectional shear of the surfaces and under an alternating external effect, has been described.

Keywords: boundary friction, friction force, shear stresses, strange attractor, Lorentz system

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INTRODUCTION

The study of the boundary friction processes that develop in nanosized tribosystems has drawn active interest from many researchers [1–4]. One of the evolving directions is the investigation of the friction of atomically smooth solid surfaces in the presence of an ultrathin film of a homogeneous lubricant between them. The interest is partially due to the applied significance of these systems, since they are used in increasing frequency to design precision equipment and instruments [6–8]. This is favored by the anomalous behavior of nanosized systems with respect to common macroscopic friction units [2, 5]. However, the majority of works are fundamental [6–8]. This is favored by the anomalous behavior of nanosized systems with respect to common macroscopic friction units. The differences are due to the fact that, if a lubricating film is a few molecular layers thick, it does not completely screen the interaction between the rubbing surfaces, the atomic relief which, in turn, has a significant effect on the microscopic structure of the lubricating film. In connection with this, the melting and solidification temperatures of ultrathin films differ from those of bulk greases. These films may be in several structural states, each of which is characterized by a specific kinetic mode of dynamic friction. Transitions between these structural states during motion yield a nonmonotonic dependence of the friction force on the velocity, etc. One of the specific features is the stick–slip mode of motion, which is not observed during hydrodynamic friction, but is typical of systems with dry friction [1, 2].

The acquired experimental data are sufficient to form a basis for a large number of theoretical studies that describe to a definite extent the boundary friction processes [6, 7, 9–12]. However, since the slightest variation in both the internal (type of lubricant [2], structure of the friction surfaces [13], pressure, etc.) and the external (load applied to the surfaces, shear rate, and type of tribosystem) parameters may have a critical effect on the nanosized systems, the full range of current views cannot be united under a universal boundary friction theory. This problem is complicated by the fact that the direct observation of the friction processes becomes more difficult due to the microscopic scale of the studied object [5, 14]. Nevertheless, important parameters, such as the thickness of the lubricating film or the number of monomolecular lubricating layers, temperature, external load, effective viscosity, and the elastic and viscous components of the shear stresses, are recorded and measured in experiments [5]. Because of this, a large number of phenomenological models have been developed recently, one of which was refined in [15–17].

A synergetic representation of the boundary friction processes [15–17] makes it possible to describe the nontrivial behavior of an ultrathin lubricating film clamped between two solids that are in relative motion. The model is based on a system of three differential equations for the stresses, strains, and lubricating film temperature. Using this model, hysteresis phenomena have been described [15], the fractal characteristics of time stress series have been studied [16], and the periodic stick–slip mode has been described under stochastic conditions [17]. However, in these works, the problem of the occurrence of oscillations in the system in the deterministic chaos mode that, according to the structure of the basic equations, can be observed in the synergetic representation has not been studied. The results of these works are presented in more detail in review [18]. However, they do not answer the question regarding which mechanical tribosystem the initial system of equations corresponds to. The present work is a continuation of [15–17] (see also [18] and references in it) and is aimed at eliminating the abovementioned drawbacks.

BASIC EQUATIONS

The system of the basic equations for the stresses, strains, and lubricating film temperature is as follows [15–18]:

$$\dot{\sigma} = -\sigma + g\varepsilon, \tag{1}$$

$$\tau\dot{\varepsilon} = -\varepsilon + (T - 1)\sigma, \tag{2}$$

$$\delta\dot{T} = (T_e - T) - \sigma\varepsilon + \sigma^2, \tag{3}$$

where σ is the shear component of the stresses that arise in the lubricating film, ε is the shear component of the relative strains, T is the lubricating film temperature, and T_e is the friction surface temperature. All of these parameters are dimensionless, which makes it possible to carry out a qualitative analysis without considering the characteristics of a given tribosystem [18]. The constant $g < 1$, which is numerically equal to the ratio of the lubricant shear modulus to its characteristic value [18], and the following parameters have been introduced here:

$$\tau = \frac{\tau_\varepsilon}{\tau_\sigma}, \quad \delta = \frac{\tau_T}{\tau_\sigma}, \tag{4}$$

where τ_σ, τ_ε, and τ_T are the times of relaxation of the stresses, strains, and temperature, respectively.

It has been shown in [15–17] that the zero stationary stresses σ_0 correspond to a solid-like lubricant structure, while at $\sigma_0 \neq 0$, the lubricant melts and transits to a fluid-like state. One of the reasons for this is that, according to the Hersey–Stribeck curve generalized for the boundary mode [19], the growth of the viscous stresses σ_v increases the relative shear rate of the rubbing surfaces as follows [18]:

$$V = \sigma_v \frac{h}{\eta_{\text{eff}}}, \tag{5}$$

where h is the lubricating film thickness and η_{eff} is the effective viscosity. Since the stresses σ are the sum of the viscous and elastic components [16, 18] and the viscous stresses dominate in a fluid-like lubricating film, with increasing σ, the velocity of motion of the surfaces being sheared increases, which corresponds to the kinetic mode of sliding and the fluid-like lubricant structure. At $\sigma = 0$, the friction surfaces do not move, which corresponds to their stick due to the solidification of the interface layer. These conclusions are confirmed by both theoretical results [6, 20] and numerous experimental data [2–4, 19].

Let us consider the stationary mode when the derivatives in Eqs. (1)–(3) are zero and the lubricant parameters remain unchanged with time. In this case, if the temperature of the friction surfaces T_e is less than the critical temperature

$$T_{c0} = 1 + g^{-1}, \tag{6}$$

the stationary value of the shear stresses $\sigma_0 = 0$ is realized. If $T_e > T_{c0}$, one of the following values corresponding to sliding sets depending on the initial conditions:

$$\sigma_0^{\pm} = \pm\sqrt{\frac{gT_e - (g+1)}{1-g}}. \tag{7}$$

Since the melting and solidification temperatures are the same as the value of the temperature T_{c0} (Eq. (6)), this case corresponds to the model of the second-order phase transition, as opposed to the first-order transition for which these temperatures are different [12].

Previously, we studied the cases when one of the relaxation times in the basic equations was assumed to be short [17, 21]. Then, system of equations (1)–(3) is reduced to a two-parameter dissipative model. In this model, in the deterministic case, a stick–slip mode of motion may occur only at the initial stage and the system of equation presents a transient mode. However, in experiments on boundary friction, a stable stick–slip mode is observed fairly often [2]. Since system of equations (1)–(3) formally coincides with the Lorentz system, the stick–slip mode can be described, in particular in a strange attractor system. According to the Ruhel–Takkens theorem, this requires three degrees of freedom, i.e., all of the relaxation times in (1)–(3) should have nonzero values [22, 23].

The stick–slip mode was described in the thermodynamic model [12, 24]. Let us consider that it can be also achieved using the synergetic approach, which has been refined in the present work. In the general case, when all of the relaxation times in system (1)–(3) have nonzero values, it is reduced to the following third-order differential equation:

$$\dddot{\sigma} - \ddot{\sigma}\left(\frac{\dot{\sigma}}{\sigma} - 1 - \frac{1}{\tau} - \frac{1}{\delta}\right) - \frac{\dot{\sigma}}{\tau}\left(\frac{\dot{\sigma}(\tau+1)}{\sigma} - \frac{\sigma^2 + 1 + \tau}{\delta}\right) - \frac{\sigma\left(g\left(T_e + \sigma^2 - 1\right) - \sigma^2 - 1\right)}{\delta\tau} = 0. \tag{8}$$

In Eq. (8), according to (5), the stresses σ are proportional to the relative velocity of motion of the rubbing surfaces and, therefore, $\dot{\sigma}$ is the acceleration.

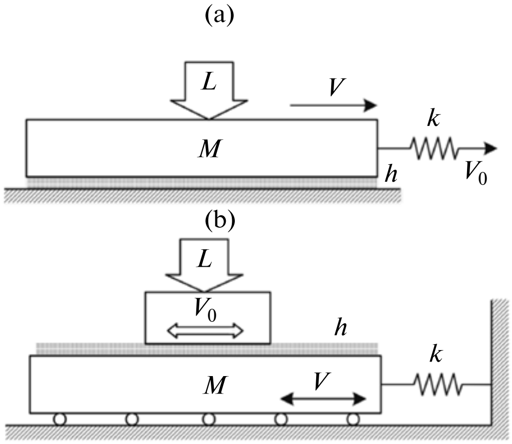

Fig. 1. Mechanical analogs of two types of tribosystems.

KINETICS

Initial system of equations (1)–(3) is based on a description of a viscoelastic medium without considering the properties of a given tribosystem. Because of this, it can be applied to describe the boundary friction mode in various types of tribosystems. Let us consider the most widespread mechanical analogs of the two tribosystems shown in Fig. 1. Figure 1a illustrates a system that consists of a spring of the stiffness k, which is connected to a block with mass M. The block lies on a stationary smooth surface and a lubricating layer of the thickness h separates the block and surface. An auxiliary load L is applied to the block. The free end of the spring moves at velocity V_0. In the general case, for ultrathin lubricating films, the velocity of the block V and that of the spring V_0 do not coincide under boundary friction because of an oscillating pattern of the friction force that hampers the motion of the block [2, 12].

The design of the system shown in Fig. 1b is as follows. A spring of the stiffness k is connected to a block of the mass M. The block is placed on rollers whose rolling friction can be neglected. The other block lies on the first block and is driven by external forces; the velocity of the motion V_0 of the upper block varies cyclically [8, 25]. If a thin lubricating film of the thickness h separates the rubbing surfaces of the blocks, the lower block is carried along by the moving upper block and the time dependence of the velocity of its motion $V(t)$ cardinally depends on the friction mode. For example, during dry friction under heavy loads L, when the surfaces of the blocks stick together, the trajectory of the lower block is the same as that of the upper block. During hydrodynamic friction when the thickness h of a film of the melted lubricant between the surfaces of the blocks is great and the load L is light, the lower block remains stationary and the motion of the upper block heats up the lubricant.

We note that the system shown in Fig. 1a was studied both experimentally [2] and using two thermodynamic models [12, 26], which was based on the Landau phase transition theory, and the stochastic model [7]. Unlike the model presented in [26], the model developed in [12] explicitly takes into account the effect of the load L. The test rig shown in Fig. 1b was experimentally studied in [8]. A similar system was also investigated in theoretical work [25].

Let us show that, in the approach developed in this study, the behavior of the systems presented in Fig. 1 can also be qualitatively described. To do this, we will solve differential equation (8) at constant parameters. Since the behavior of the system described by Eq. (8) depends critically on the initial conditions, it is appropriate to present the solution as phase portraits. Figure 2 shows the corresponding phase portraits at different temperatures of the friction surfaces T_e. In all of the figures, the initial value of the second time derivative is $\ddot{\sigma}_0 = 0$. The initial values of $\dot{\sigma}$ and σ can be seen directly on the phase planes and correspond to the origin of the phase trajectories.

Let us consider in more details Fig. 2a. The phase portrait is characterized by the only critical point at the origin of coordinates when $\sigma = \dot{\sigma} = 0$; this point is a stable focus. In particular, at the initial value $\sigma_0 \sim 0$, the focus becomes more pronounced (trajectory *7*). For all of the phase trajectories in Fig. 2a, the corresponding dependences of the stresses σ on the time t have been plotted (Fig. 3a). It can be seen that, for trajectories *2*, *3*, *5*, and *6*, an aperiodic mode is realized in which the stresses relax to the stable value $\sigma = 0$ that correspond to a solid-like lubricant structure. Trajectories *1*, *4*, and *7* correspond to the damped oscillation mode since, unlike the above-considered aperiodic trajectories *2*, *3*, *5*, and *6*, in all of these cases, several extreme points occur on the corresponding curves $\sigma(t)$ (Fig. 3a) until equilibrium sets in. For the parameters of trajectory *7*, damping is less pronounced, since several oscillations are observed in the vicinity of the stable value $\sigma = 0$ during relaxation. In all of these cases (trajectories *1*–*7*), the moving block shown in Fig. 1 stops at some moment in time. In the case presented in Fig. 1a, this may occur at the value of the velocity $V_0 = 0$ and the initial elongation of the spring that corresponds to the initial values of the system parameters on the appropriate phase trajectories (Fig. 2a). This situation also describes the behavior of the system shown in Fig. 1b at $V_0 = 0$ when the lower block is not initially in the equilibrium state, i.e., the spring is either compressed or stretched. We note that the value $\sigma < 0$ corresponds to a negative velocity (reverse motion).

Figure 2b illustrates a phase portrait at the temperature of the friction surfaces $T_e > T_{c0}$ (Eq. (6)) at which the nonzero value of the stresses σ_0 determined by formula (7) sets in and the rubbing block moves with the

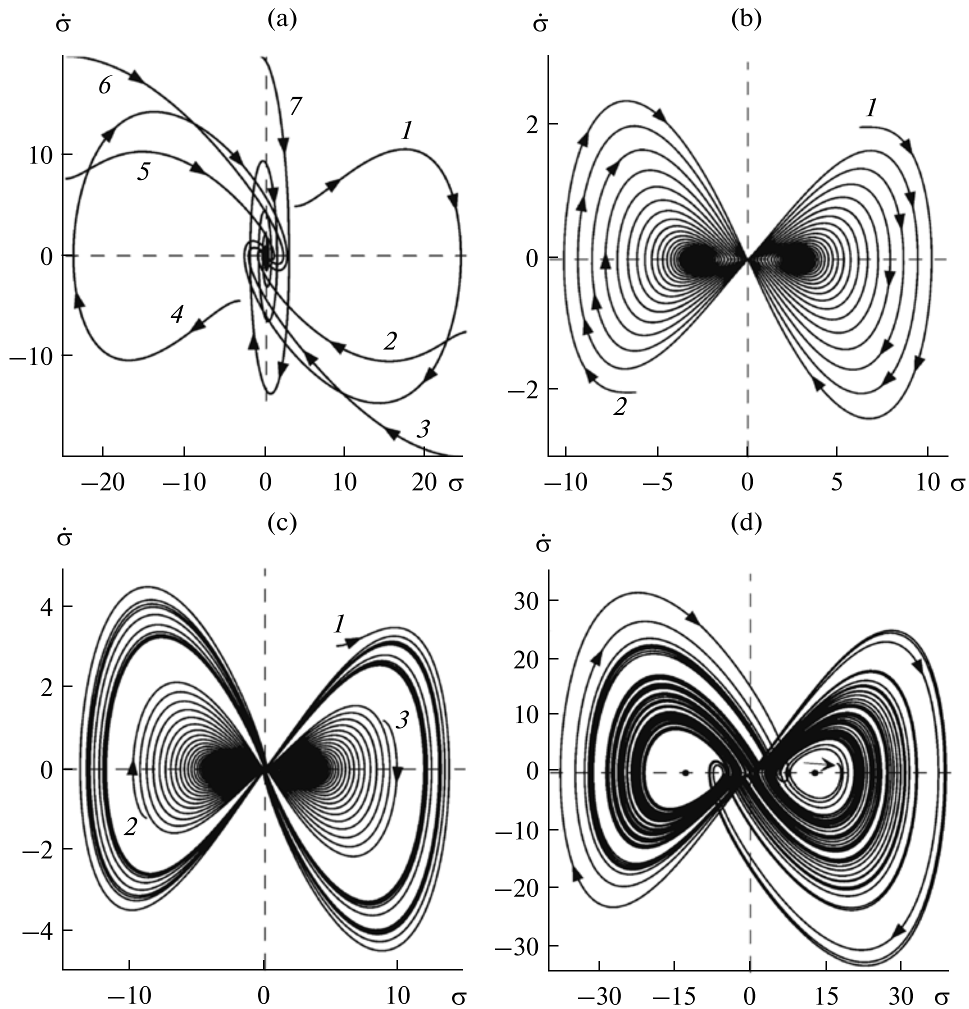

Fig. 2. Phase portraits of system obtained during numerical solution of equation (8) at parameters $g = 0.25$, $\tau = 3$, and $\delta = 95$. Figures (a)–(d) correspond to temperatures $T_e = 1$, 30, 38, and 500.

nonzero velocity (5). In the phase portrait, two critical points occur with the coordinates $(\sigma_0, 0)$ and $(-\sigma_0, 0)$ that are symmetric with regard to the origin of coordinates. Both points are equivalent and represent stable focuses around which long oscillations develop until the block begins to move with a constant velocity (Fig. 3b). We note that, in the beginning of motion, long portions $\sigma \approx 0$ exist on the trajectories $\sigma(t)$ that corresponds to the stick of the surfaces and the solid-like lubricant structure. Thus, at the initial stage, the stick–slip mode of motion is achieved [2, 25, 26] when periodic phase transitions between the structural states of the lubricant occur during friction. Because, under all initial conditions motion sets in at this stage with a constant velocity, this situation corresponds to the tribosystem presented in Fig. 1a at $V_0 \neq 0$. Trajectory *1* is appropriate to motion in the direction shown by the vector V_0 in Fig. 1a and trajectory *2* to motion in the opposite direction. We note that, at the initial value $\sigma_0 \approx 0$, the situation shown by trajectory *7* in Fig. 2a is achieved at initial stage. The difference is that some changeovers between the vicinities of the critical points occur after relaxation and the subsequent long stop of the system near the origin of coordinates[1]; the changeovers are the curls in the phase portraits at the positive and negative values of the stresses σ. Only after this does a dissipative mode of damped oscillations set in; depending on the initial conditions, the oscillations may occur at both positive and negative stresses σ.

As the temperature T_e elevates, the situation shown by the phase portrait in Fig. 2c is achieved. In this case, the set of the initial values of σ_0, $\dot{\sigma}_0$, and $\ddot{\sigma}_0$ has a critical effect on the behavior of the system. It can be seen that, at low absolute values of the stresses σ_0 and the rate of their change $\dot{\sigma}_0$, the same critical points occur

[1] The duration of a stop after relaxation can amount to over 400 units. These long stops correspond to the stop–start experiments [2] that demonstrate the realization of memory effects in the boundary lubrication mode.

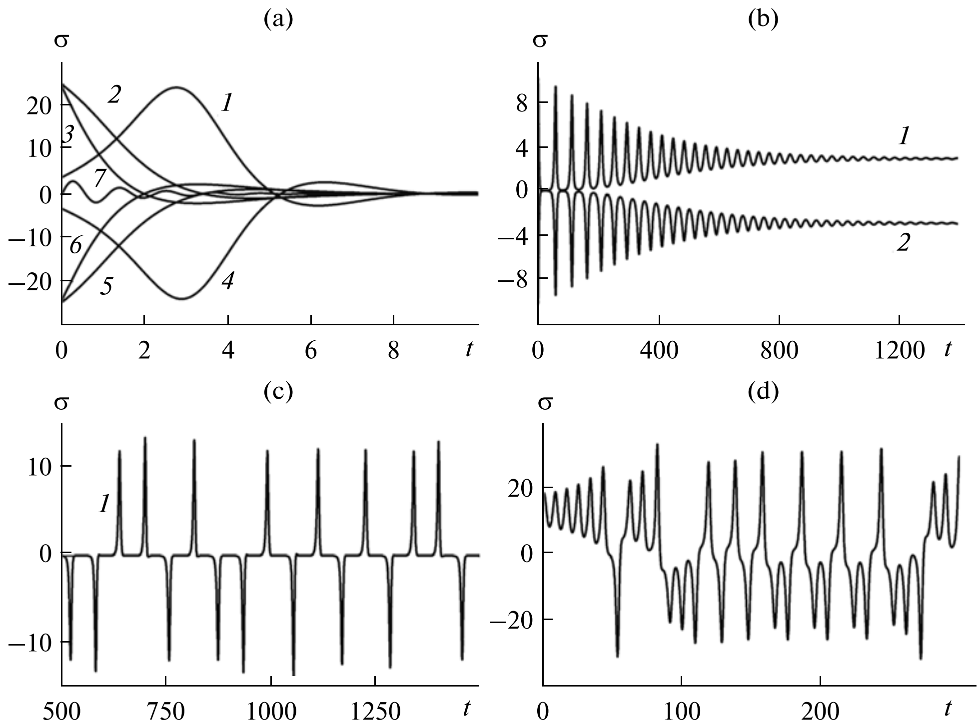

Fig. 3. Time dependences of stresses $\sigma(t)$ corresponding to numbered phase trajectories shown on appropriate panels in Fig. 2.

in the phase portrait as in Fig. 2b (trajectories *2* and *3* in Fig. 2c). However, if in the beginning of motion, the stresses σ_0 or the rate of their change $\dot{\sigma}_0$ exceed critical values, a chaotic mode sets in. In this case, no stationary state is achieved with time, but constant phase transitions between the solid- and fluid-like lubricant states occur (trajectory *1*). It can be seen in Figs. 2c and 3c that this mode is not periodic in time and represents a strange attractor, i.e., a realization of the deterministic chaotic mode. Since, according to the data in Fig. 3c, the velocity of the motion of the rubbing block constantly changes its sign, this situation describes the behavior of the system shown in Fig. 1b in which the direction of motion alters due to the external effect. However, at high values of V_0, the reverse mode under consideration also occurs in the systems whose mechanical analog is presented in Fig. 1a. This is favored by the fact that, at a high sliding velocity V_0 of the free spring end, during the period of the stick of the surfaces when $\sigma = 0$ and the lubricant is in the solid-like state, the spring has time to get strongly stretched. During the subsequent melting of the lubricant, the block slips over a long distance due to a high elastic force of spring stretching $k\Delta x$ (Δx is the elongation of the spring) and the spring is compressed. This yields the alteration of the elastic force direction and, therefore, the block continues to move oppositely for a period of time. However, since the velocity V_0 does not change its sign over time, the direction of motion will again switch to positive, which is confirmed in Fig. 3c. These reverse modes have been experimentally observed [2]. We note that, in this phase portrait, the mode shown by trajectory *7* in Fig. 2a may occur, as in the previous phase portraits. However, unlike the previous cases, after relaxation and the long stop of the system, the strange attractor mode sets in it. After the stop and the subsequent beginning of motion, several curls appear in the phase portrait, the radii of which are much greater than those of the curls shown in Fig. 2c. This is explained by the fact that, since the surfaces have long been in the stick state, the driving elastic force $k\Delta x$ increases manifold during spring stretching. Just after melting, this leads to higher values of acceleration and the velocity of motion of the block, i.e., the stresses σ and the derivative $\dot{\sigma}$ grow.

Figure 2d is plotted at a fairly high value of the temperature T_e; the arrow in the right part of the figure shows the point that corresponds to the origin of the trajectory (initial conditions during the solution of Eq. (8)). In this figure, two symmetric critical points occur and are shown by the points on the abscissa axis. Unlike Fig. 2c in which the stable focuses are presented, these points are unstable focuses[2]. Therefore, under any initial conditions, the friction mode to

[2] The abscissas of the critical points in Fig. 2 are determined according to relations (7).

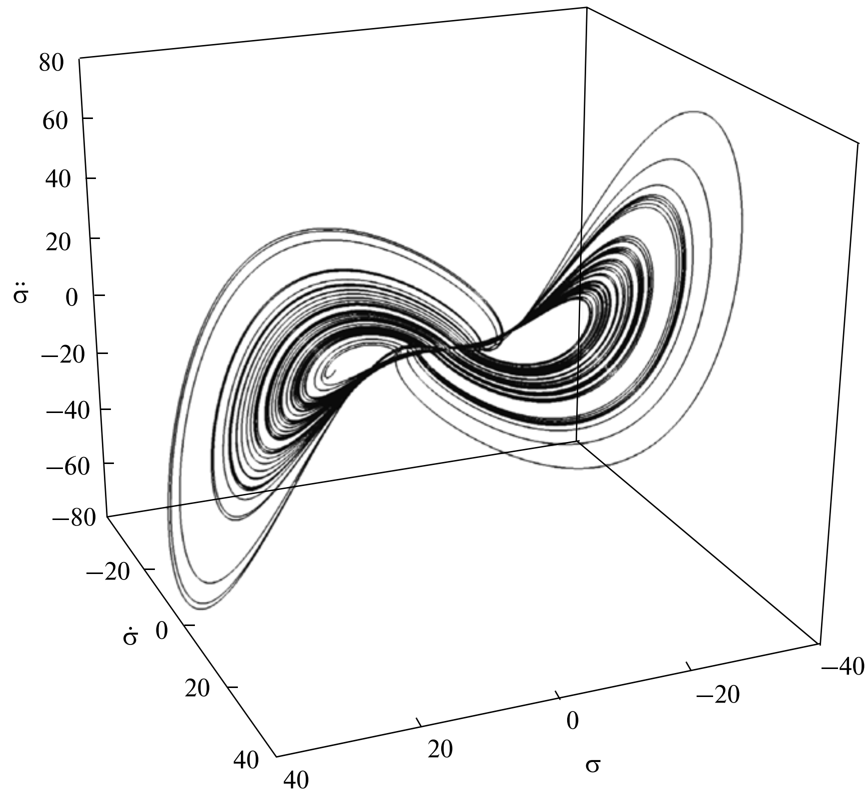

Fig. 4. Three-dimensional phase trajectory shown in projection in Fig. 2d. For clearness, positions of butterfly wings are transposed as compared to Fig. 2d. In Fig. 4, entrance into attractor is seen on left-hand side.

which the realization of the strange attractor on the phase plane corresponds is set in Fig. 2d. We note that the phase trajectory is shaped as a Lorentz strange attractor [23, 27] and, in this case, the mode that occurs is characterized by a more pronounced stochastic behavior than that shown in Fig. 2c. This is due to the fact that the temperature T_e in the initial system of equations represents the external effect, i.e., at a higher temperature, the system possess a greater stored energy, which, in our case, appears as an increase in stochasticity. The case shown by trajectory *7* in Fig. 2a is also typical of the parameters of the figure under consideration. Here, after a long stop and subsequent motion with a higher velocity, the strange attractor mode presented in Fig. 3d sets in. For clearness, the phase trajectory, which, in Fig. 3d, is only shown in three-dimensional form in the coordinates $\sigma-\dot{\sigma}-\ddot{\sigma}$, is illustrated in Fig. 4. The plot is a Lorentz butterfly, which is well known in chaos theory [23, 27]. However, since system (1)–(3) is only reduced formally to the Lorentz system, there are visible differences. We would like to note that the described mode can be observed more often in the tribological system shown in Fig. 1b, while the realization of this mode in the system presented in Fig. 1a requires that specific conditions should be satisfied (see the description of Fig. 2c). Thus, cardinally different friction modes have been distinguished in Fig. 2 and shown to occur in various types of tribosystems. This means that, although the model is qualitative due to the use of a number of assumptions during the derivation of the basic equations [16], it is universal, since the characteristics of a given mechanical tribosystem are not taken into account when deriving the equations.

CONCLUSIONS

This work presents the subsequent development of the synergetic model, which describes the state of an ultrathin lubricating film clamped between two atomically smooth solid surfaces during boundary friction. It has been shown that the use of this model can make it possible to describe the behavior of various types of tribosystems. The stick–slip mode frequently observed in experiments has been depicted. In this mode, consecutive transitions between the structural states of the lubricant occur. This work makes it possible to extend the results of the previous studies obtained in the synergetic model, since the described stick–slip mode has a deterministic nature. This has not been shown previously, but is observed in numerous experiments on studying the boundary friction processes. It has been found that, as the temperature of the friction surfaces elevates, the stochasticity in the system grows. When the temperature exceeds a critical value, the system follows the mode described by the Lorentz attractor. In the wide range of parameters, the reverse motion of the rubbing surfaces occurs. Our results agree qualitatively with known experimental data. It has been shown that, in all of the considered modes, a similar transient mode occurs in a definite range of initial conditions. This mode involves damped oscillations and the subsequent stick of the surfaces together for a long time, after which a stationary mode of friction sets in. If the initial conditions fall outside the above-mentioned range, the transient mode is governed by the system parameters.

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DESIGNATIONS

σ—shear component of total stresses; ε—shear component of total strains; T—temperature of lubricating film; $\dot{\sigma}$, $\dot{\varepsilon}$, $\dot{T}$—time derivatives of corresponding variables; T_e—temperature of friction surfaces; τ_σ, τ_ε, and τ_T—times of relaxation of stresses, strains, and temperature; $\tau = \tau_\varepsilon/\tau_\sigma$ and $\delta = \tau_T/\tau_\sigma$—ratios of times of relaxation; $g < 1$—ratio of shear modulus of lubricant to its characteristic value; h—lubricating film thickness; σ_v—viscous component of shear stresses; η_{eff}—effective viscosity of lubricant; V—relative velocity of shear of rubbing surfaces; T_{c0}—melting temperature of lubricant; σ_0—stationary value of stresses; $\sigma_0^{\pm}$—symmetric stationary values of stresses; $\ddot{\sigma}$, $\dddot{\sigma}$—second and third time derivatives of stresses; L—external normal load applied to friction surfaces; M—mass of rubbing block; k—stiffness of spring; V_0—velocity of external drive; σ_0, $\dot{\sigma}_0$, $\ddot{\sigma}_0$—initial values of corresponding parameters; Δx—elongation of spring.

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